

| | |
|---------------|---|
| $\frac{5}{6}$ | Numerator - tells how many pieces we have. Denominator - names the family of fractions |
|---------------|---|

Figure 1.


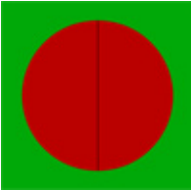
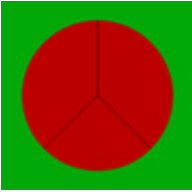
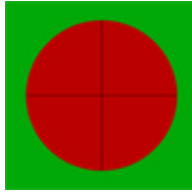
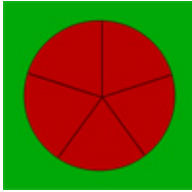
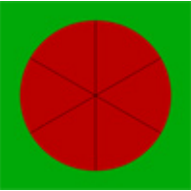
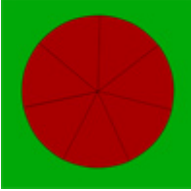

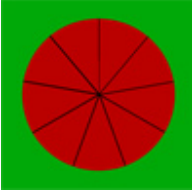
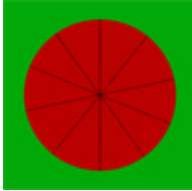
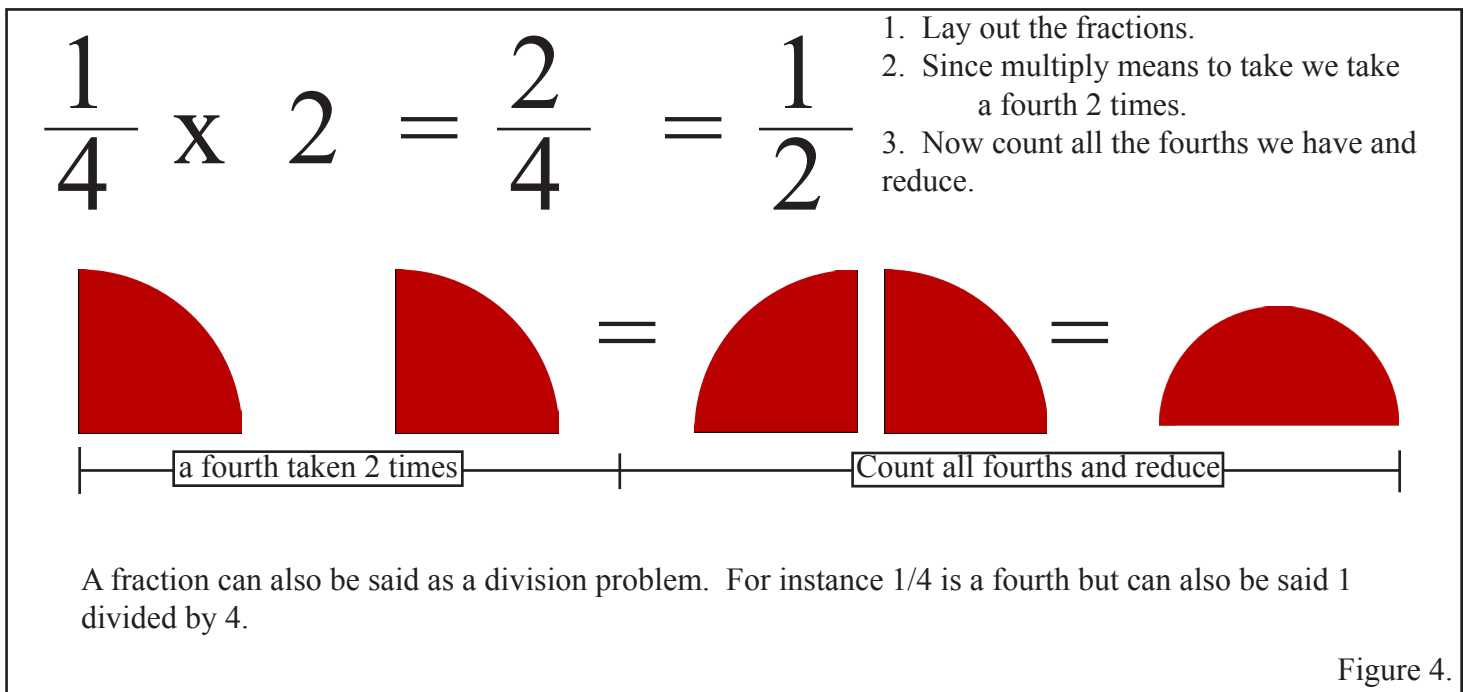
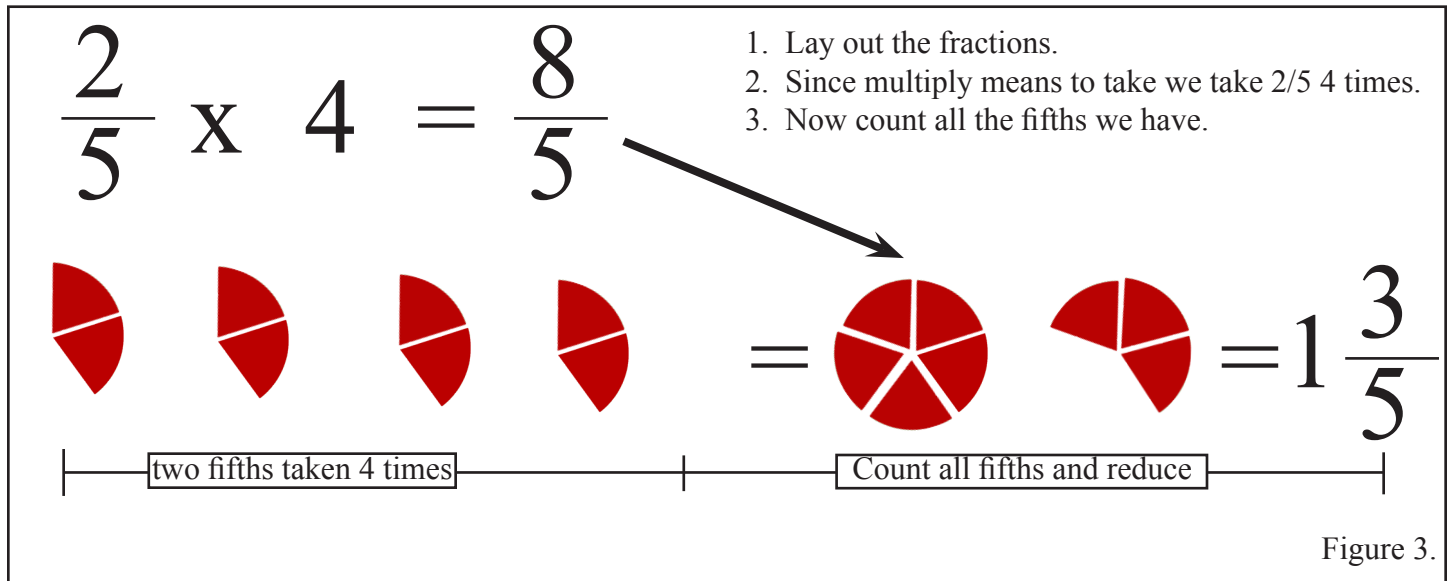
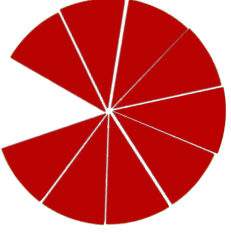
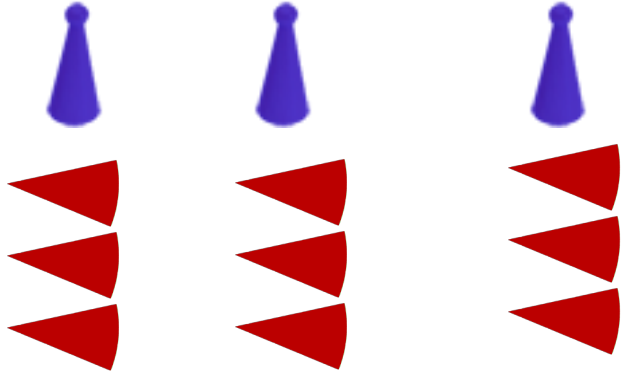
| | | | | |
|---|---|---|--|---|
|  |  |  |  |  |
| $\frac{1}{1}$ Whole | $\frac{2}{2}$ Halves | $\frac{3}{3}$ Thirds | $\frac{4}{4}$ fourths | $\frac{5}{5}$ Fifths |
|  |  |  |  |  |
| $\frac{6}{6}$ Sixth | $\frac{7}{7}$ Seventh | $\frac{8}{8}$ Eighths | $\frac{9}{9}$ Ninth | $\frac{10}{10}$ Tenths |

Figure 2.

Multiplication of a Fraction by a Whole Number.



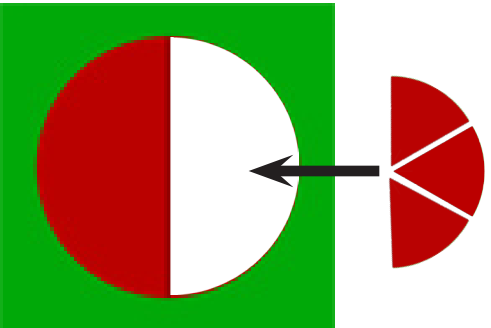
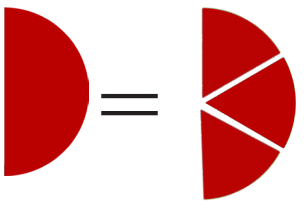
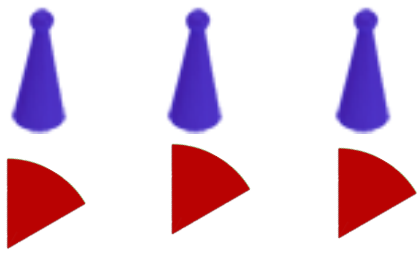
Division of a Fraction by a Whole Number.

$$\frac{9}{10} \div 3 = \frac{3}{10}$$



1. Lay out the larger fraction.
2. Place three skittles down.
3. Distribute the tenths evenly among the 3 skittles.

Your answer is what one skittle gets or three tenths.

Figure 5.


$$\frac{1}{2} \div 3 = \frac{1}{6}$$




1. Lay out a half and three skittles.
2. To divide evenly among the 3 skittles we need to find what goes into a half 3 equal times (what fits into the white section of the fraction? a sixth)
3. Replace the half with three sixths.
4. Distribute evenly among the skittles.

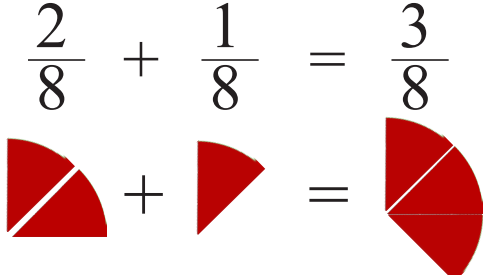
Figure 6.

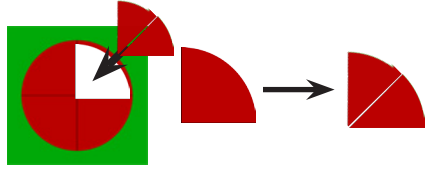
Addition and Subtraction of Fractions Having Different Demonminators.

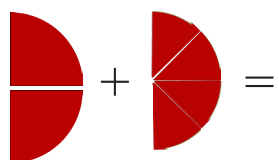
Figure 7

$$\frac{1}{4} + \frac{1}{8} =$$


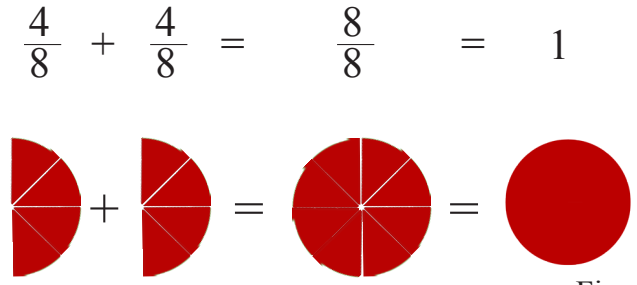
1. Lay out the fractions.
2. Try to place the fourth into the empty space in the eighths circle. Doesn't fit. Try placing eighths into the empty fourths circle. It works. Substitute the fourth for two eighths.
3. Using 2/8 rather than 1/4 add.

$$\frac{2}{8} + \frac{1}{8} = \frac{3}{8}$$


$$\frac{1}{4} \rightarrow \frac{2}{8}$$


$$\frac{2}{4} + \frac{4}{8} =$$


1. Lay out the problem.
2. Try to place the fourth into the empty space in the eighths circle. It works. Try placing eighths into the empty fourths circle. It works. Can substitute either way. Substitute 2 fourths for 4 eighths.
3. Using 4/8 rather than 2/4 add. We get 8/8 that reduces to a whole.

$$\frac{4}{8} + \frac{4}{8} = \frac{8}{8} = 1$$


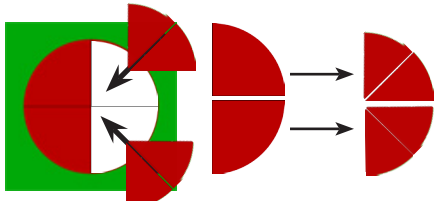
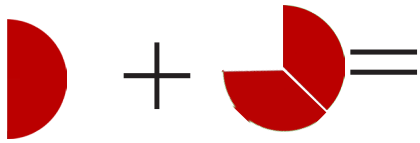
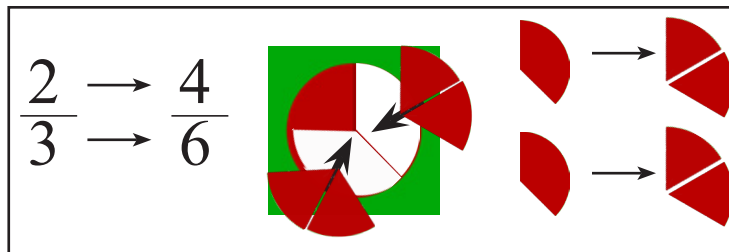
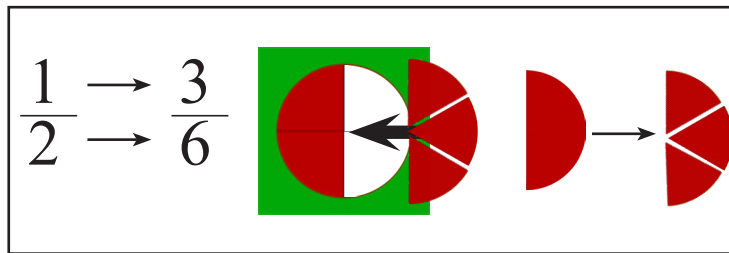
$$\frac{2}{4} \rightarrow \frac{4}{8}$$


Figure 8

$$\frac{1}{2} + \frac{2}{3} =$$


1. Lay out the fractions.
2. Because we cannot fill the thirds frame with a half and vice versa, we look for a new denominator that goes into a third and a half evenly. First one we come to are sixths.
3. Substitute a half for 3 sixths
4. Substitute 2 thirds for 4 sixths.
5. Using 3/6 rather than 1/2 and 4/6 rather than 2/3 add.

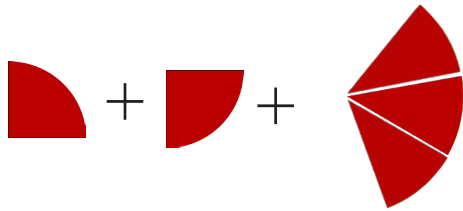


$$\frac{3}{6} + \frac{4}{6} = \frac{7}{6} = 1\frac{1}{6}$$



Figure 9

$$\frac{1}{4} + \frac{1}{4} + \frac{3}{9} =$$



1. Lay out the fractions.
 2. Since we see that a fourth plus a fourth is a half we combine.
 3. Because we cannot fill the ninths frame with a half and vice versa, we look for a new denominator that goes into 3 ninths and a half evenly. First one we come to are sixths.
 4. Substitute 3 ninths for 2 sixths.
 5. Substitute the half for 3 sixths.
- Using 3/6 rather than 1/2 and 4/6 rather than 2/3 add.

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\begin{array}{l} \frac{3}{9} \rightarrow \frac{2}{6} \\ \frac{3}{9} \rightarrow \frac{2}{6} \end{array}$$

$$\begin{array}{l} \frac{1}{2} \rightarrow \frac{3}{6} \\ \frac{1}{2} \rightarrow \frac{3}{6} \end{array}$$

$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

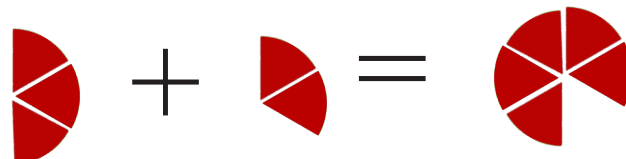
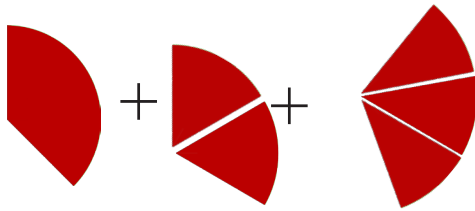
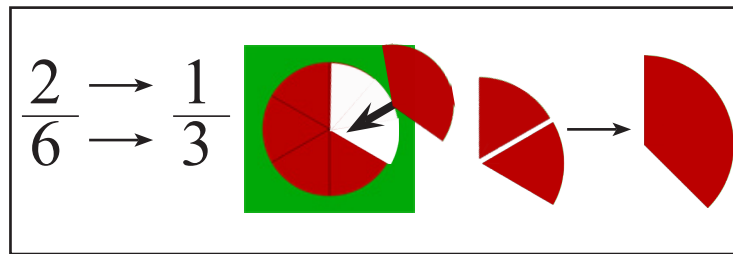
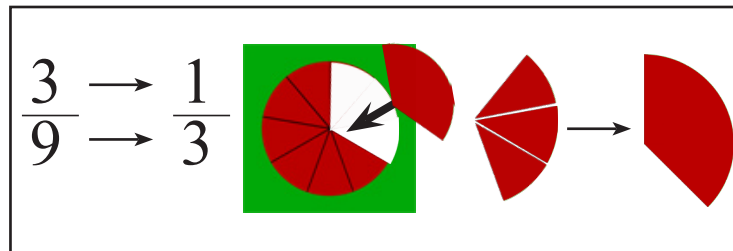


Figure 10 .

$$\frac{1}{3} + \frac{2}{6} + \frac{3}{9} =$$



1. Lay out the fractions.
2. Since we see that 2 sixths and 3 ninths can reduce to a third each substitute them.
3. Since we can reduce the fractions to the same family we just have to add them together.



$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$$

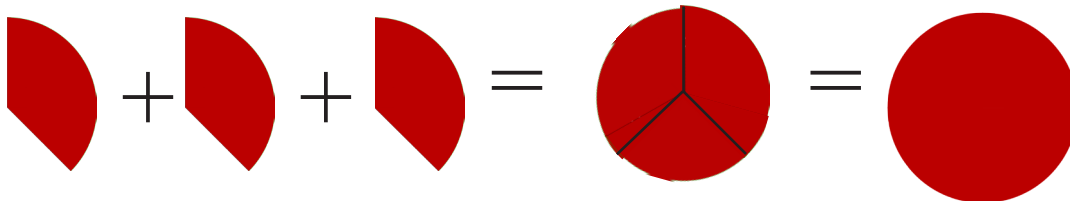
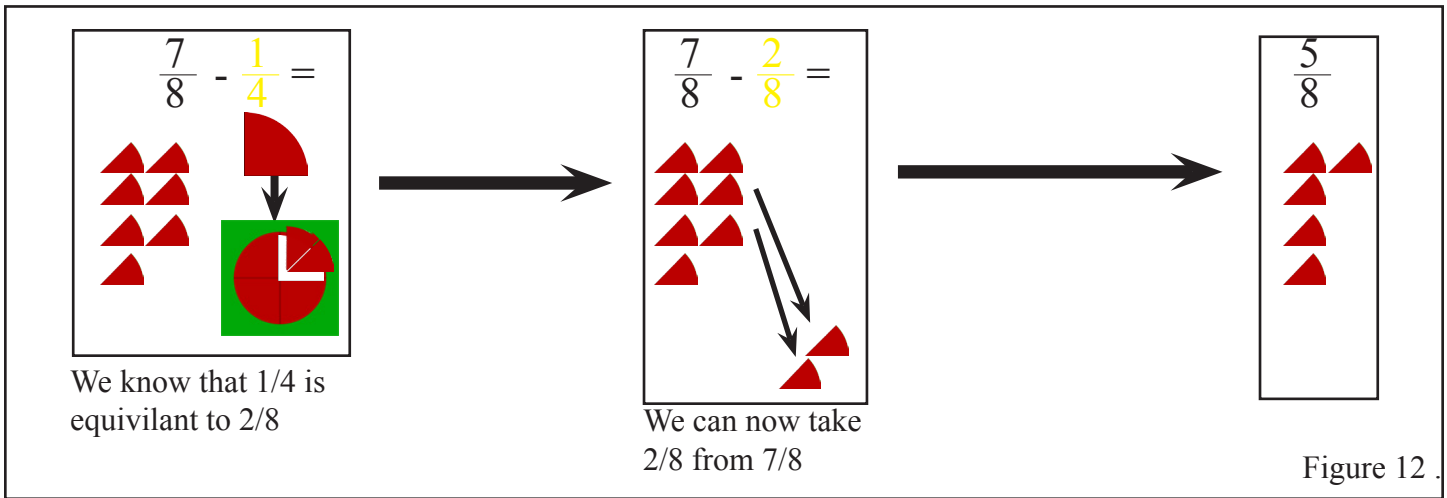
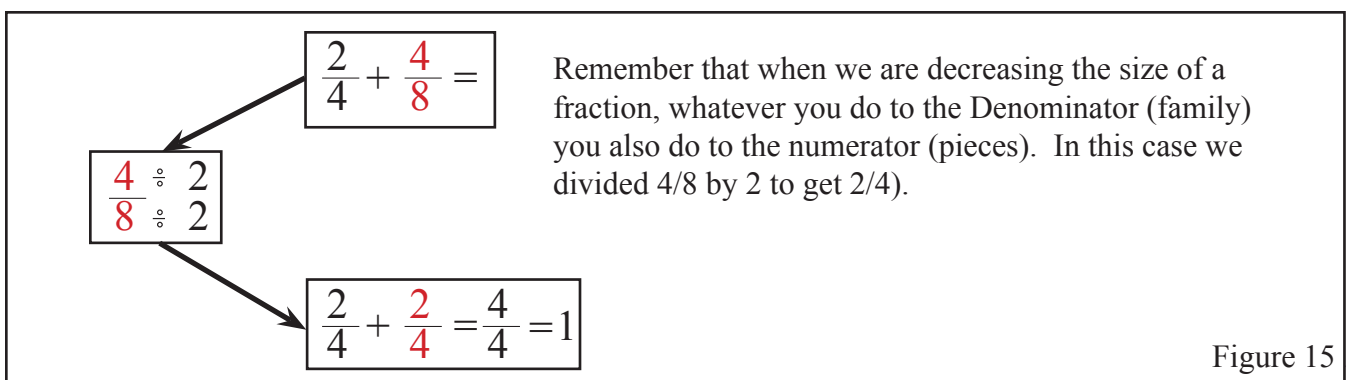
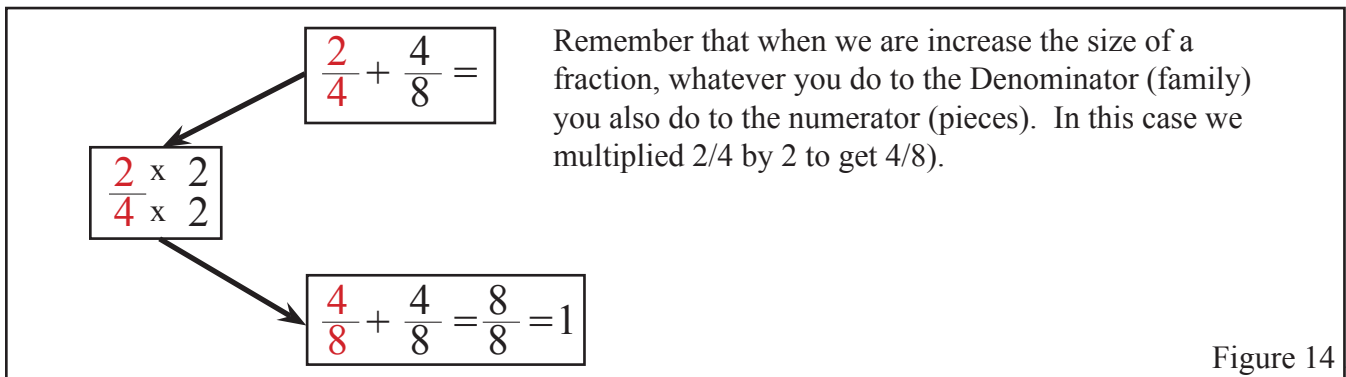
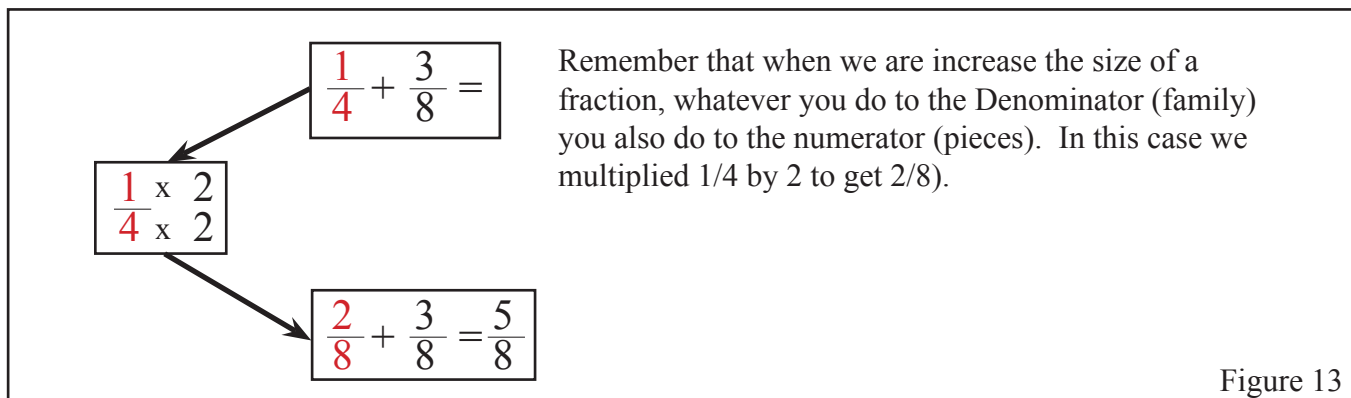


Figure 11 .



Step to Abstraction regarding Operations with Fractions.



| | |
|----------------|--|
| $\frac{2}{5}$ | proper fraction - a fraction that can not be broken down. It is proper as it is.. |
| $\frac{5}{5}$ | apparent fraction - When you can change a fraction into a whole number. |
| $\frac{8}{5}$ | improper fraction - when you can change in more than enough pieces to create a whole number and its fractional part. |
| $1\frac{3}{5}$ | mixed number - when you have a fractional part and a whole number. |

Figure 16.

Multiplication of a Whole Number by a Fraction.

$2 \times \frac{1}{2} = 1$

1. Lay out the fractions.
2. We need to take a half from 2 wholes. Put them together to get the answer of 1.

Figure 17.

$2 \times \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

1. Lay out the fractions.
2. We need to take a fourth from 2 wholes. Put them together. We get 2 fourths which simplifies to a half.

Figure 18.

$2 \times \frac{1}{5} = \frac{2}{5}$

1. Lay out the fractions.
2. We need to take a fifth from 2 wholes. Put them together. We get 2 fifth.

Figure 19.

$2 \div \frac{1}{2} = 4$

1. Lay out fraction and split skittle in half.
2. Give 2 units to half a skittle.
3. We must give 2 more units to the other half of the skittle.
4. Add the 2 halves together to get your answer.

Figure 20.

$2 \div \frac{2}{2} = 2$

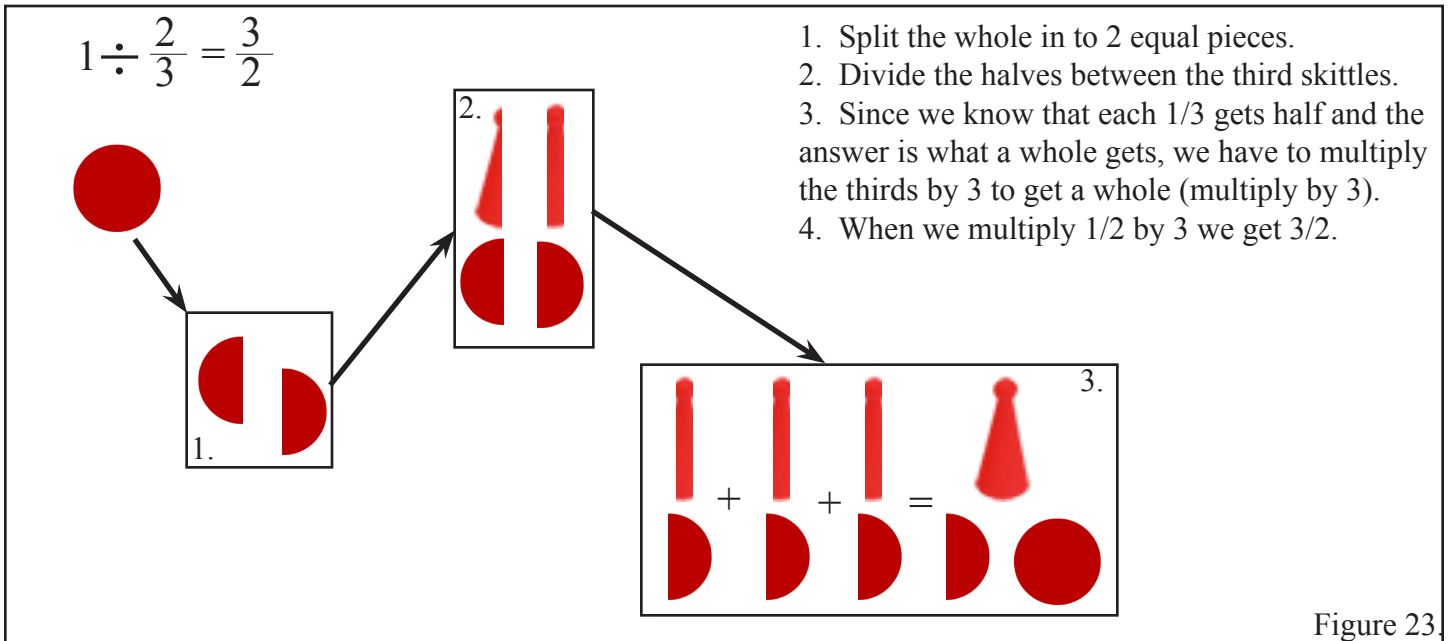
1. Lay out fraction and split skittle in half.
2. Give 1 units to the 2 halves of the skittle.
3. To figure out what 1 gets we add the half skittles together.
4. Add the 2 halves together to get your answer.

Figure 21.

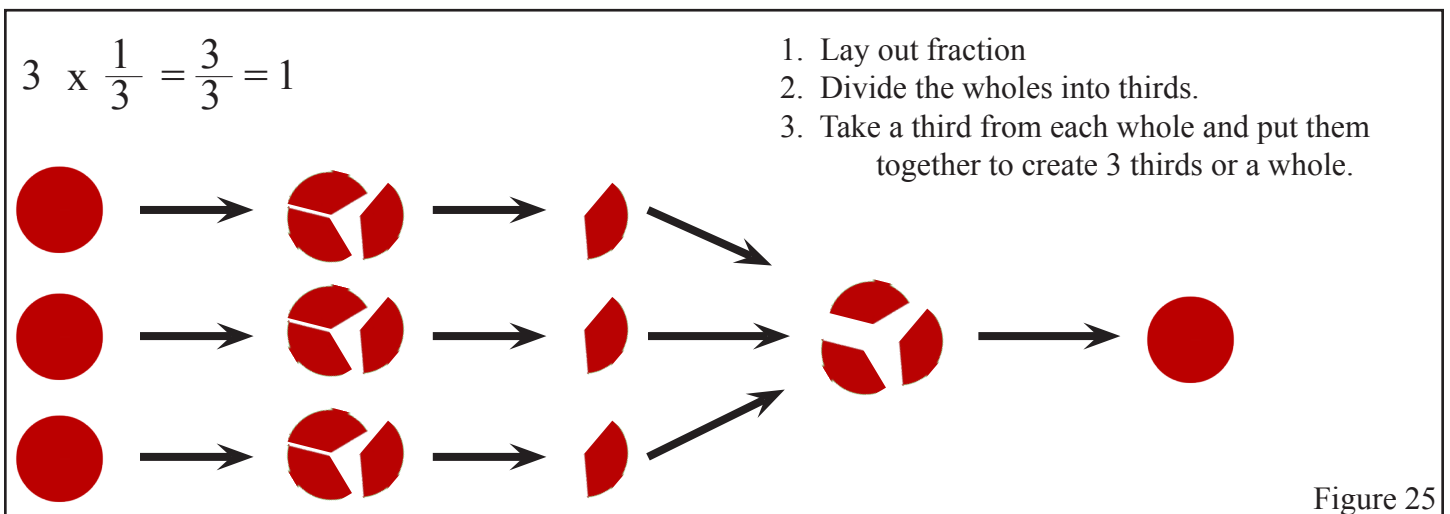
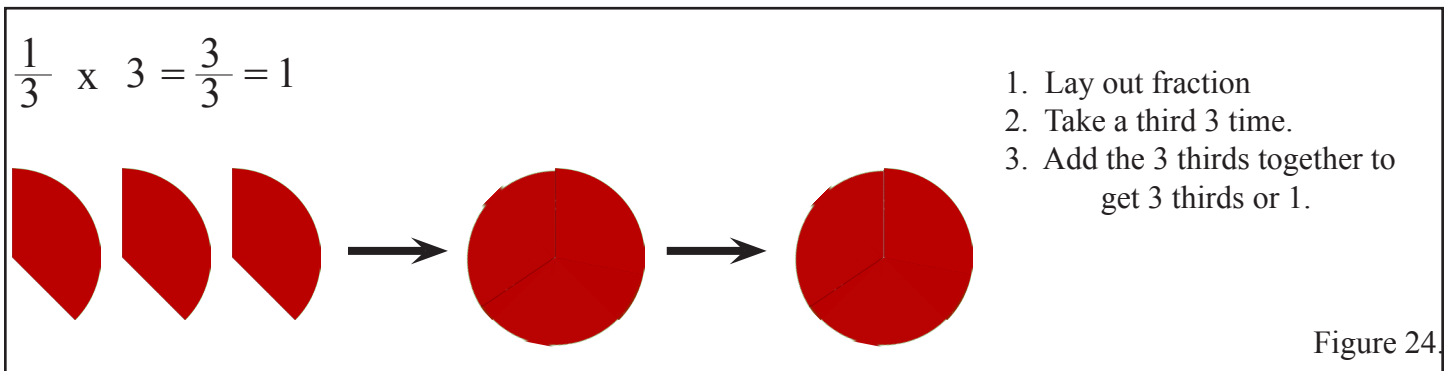
$1 \div \frac{2}{2} = 1$

1. Lay out fraction and split skittle in half.
2. Divide the whole into halves.
3. Distribute the halves to the skittles.
4. Add the 2 halves together to get your answer.

Figure 22.



Multiplication of a Fraction by a Fraction



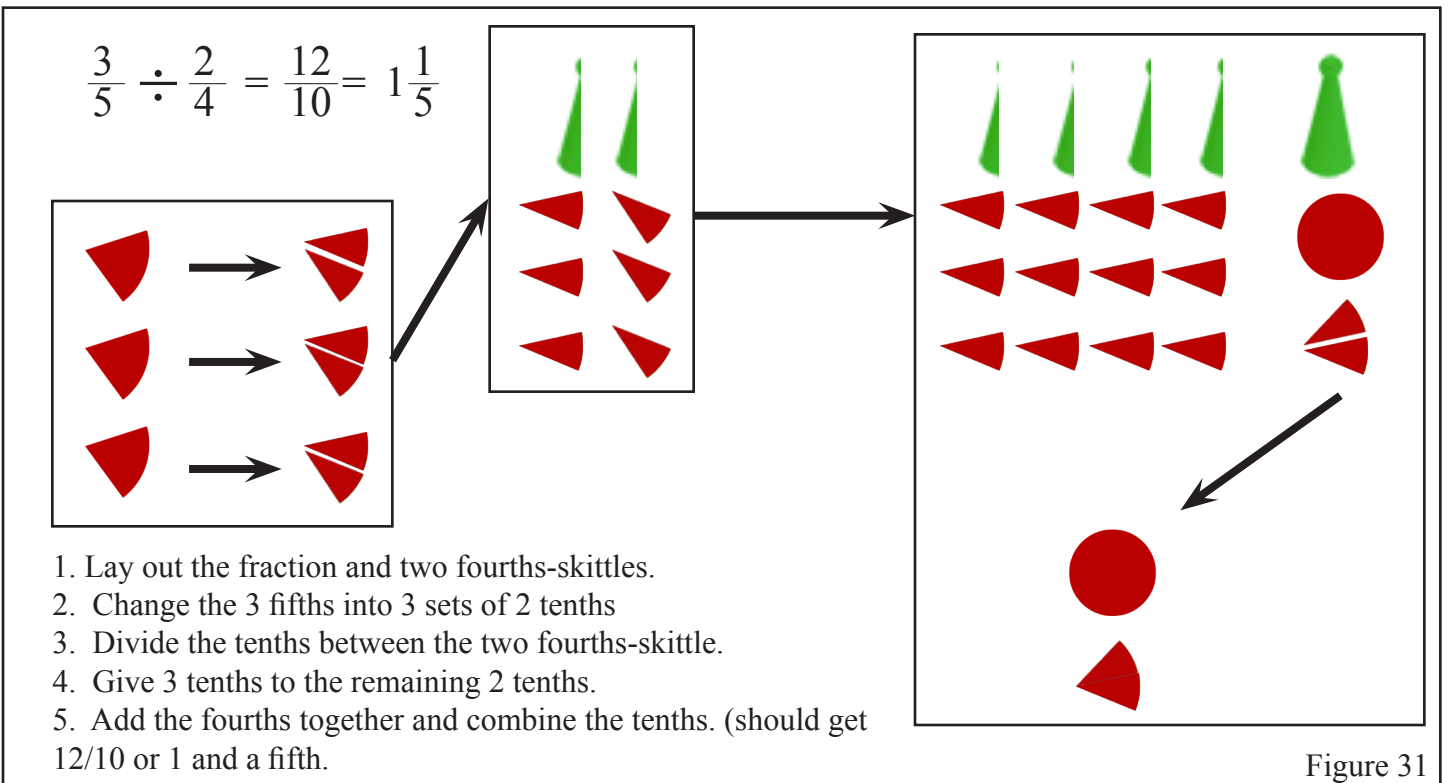
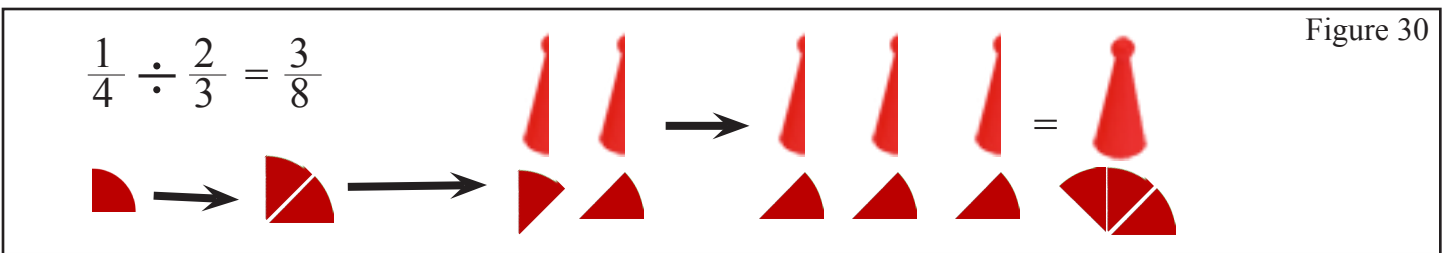
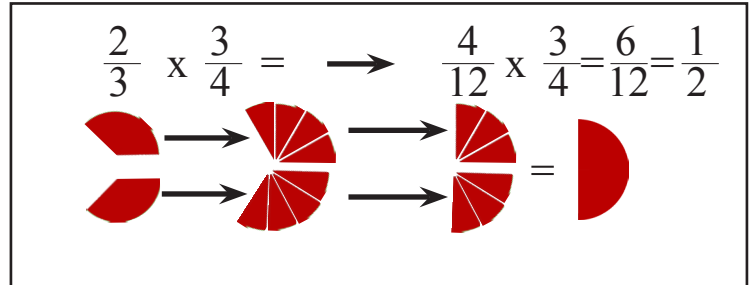
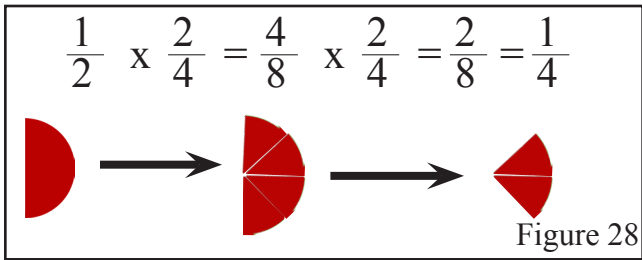
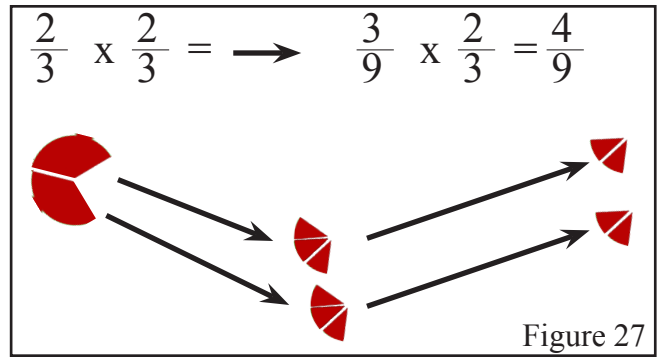
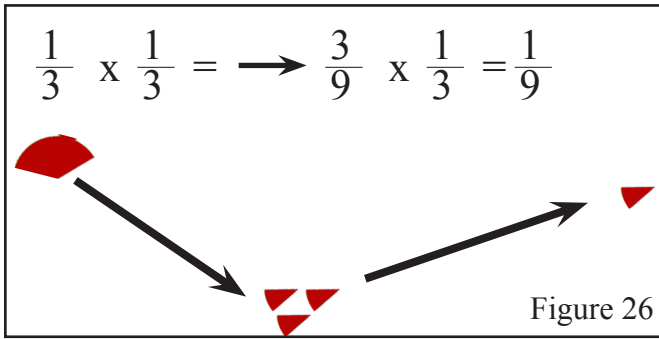
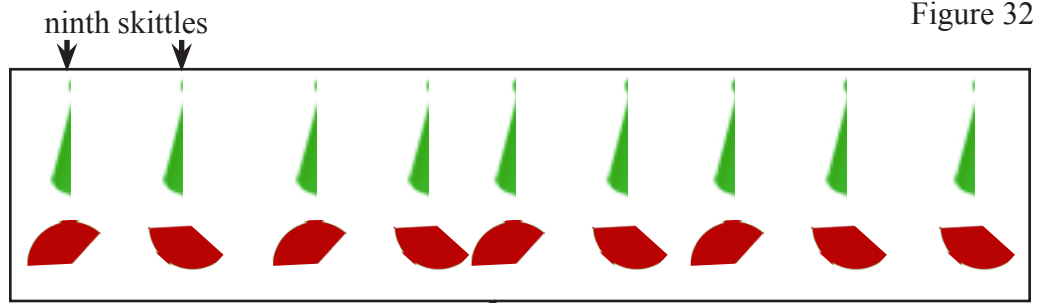
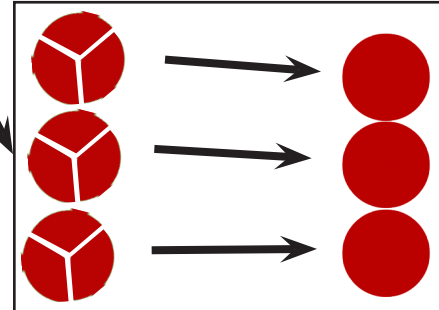


Figure 32

$$\frac{2}{3} \div \frac{2}{9} = \frac{9}{3} = 3$$



1. Lay out the fraction and two ninths-skittles.
2. Distribute a third to each skittle.
3. Give a third to the remaining 7 ninths-skittles.
5. Add the fourths together and combine the tenths. (should get 12/10 or 1 and a fifth).



Passage to Abstraction:

$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

$$\frac{2}{3} \times \frac{2}{3} = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$$

$$\frac{2}{1} \times \frac{1}{2} = \frac{2}{1} \times \frac{1}{2} = \frac{2}{2} = 2$$

Rule: When multiplying a fraction by a fraction you multiply the numerator by the numerator and the denominator by the denominator.

$$\frac{1}{3} \div \frac{1}{3} = \frac{1}{3} \times \frac{3}{1} = \frac{3}{3} = 1$$

$$\frac{2}{3} \div \frac{2}{3} = \frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$$

$$\frac{2}{1} \div \frac{1}{2} = \frac{2}{1} \times \frac{2}{1} = \frac{4}{1} = 4$$

Rule: When dividing a fraction by another, you multiply the first fraction by the inverse of the second.

$$\frac{3}{5} \div \frac{2}{4} = \longrightarrow \frac{3}{5} \times \frac{4}{2} = \frac{12}{10} = 1 \frac{1}{5}$$